Math 1B Quiz 1 Version 4

Wed Apr 18, 2018

NAME YOU

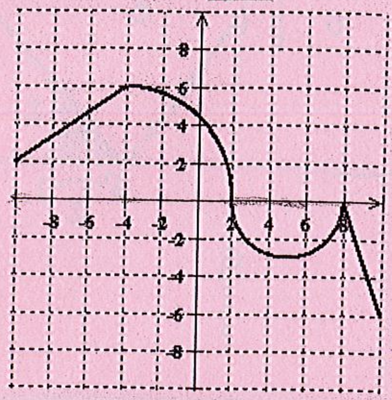
SCORE: _____ / 30 POINTS 13½ + ___ ← GREENSHEET QUIZ SCORE

1.	No calculators allowed
2.	Simplify all answers unless stated otherwise
3.	Show proper calculus level work to justify your answers

The graph of function f is shown on the right.

SCORE: $3\frac{1}{2}$ **/ 4 PTS**

The graph consists of a diagonal line, arcs of 2 circles, then another diagonal line.



[a] Evaluate $\int_{-10}^{10} f(x)\,dx$.

NOTE: You must show the arithmetic expression that you used to get your answer.

$\int_{-10}^{-4} f(x)\,dx + \int_{-4}^{2} f(x)\,dx - \int_{2}^{8} f(x)\,dx - \int_{8}^{10} f(x)\,dx$

$\left(6(2) + \frac{6(4)}{2}\right) + \left(\frac{1}{4}\pi 36\right) - \left(\frac{1}{2}\pi 9\right) - \left(\frac{2(6)}{2}\right)$ ½ ½ ½

$24 + 9\pi - \frac{9}{2}\pi - 6 = 18 + \frac{9\pi}{2} = \boxed{\frac{36+9\pi}{2}}$ ½

[b] Evaluate $\int_{8}^{-10} f(x)\,dx$.

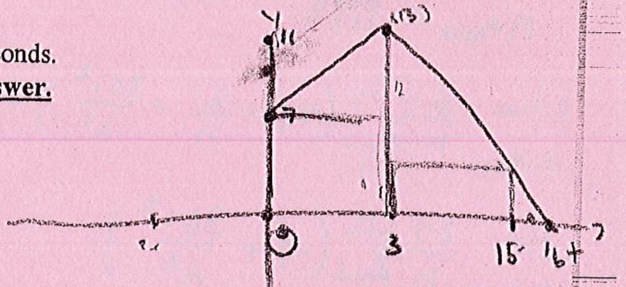
$\int_{2}^{6} f(x)\,dx = ?$

$-\int_{-10}^{8} f(x)\,dx = -\left(24 + 9\pi - \int_{2}^{8} f(x)\,dx\right) = -\left(24 + 9\pi - \frac{9\pi}{2}\right)$

$= -\left(24 + \frac{9\pi}{2}\right) = \boxed{-\left(\frac{48+9\pi}{2}\right)}$ 1 ½

A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 2t+7, & 0 \le t \le 3 \\ 16-t, & 3 \le t \le 15 \end{cases}$.

SCORE: $1\frac{1}{2}$ **/ 5 PTS**



[a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 15$ seconds.

NOTE: You must show the arithmetic expression that you used to get your answer.
You may only use techniques discussed in sections 5.1 and 5.2.

$\int_{0}^{15} f(t)\,dt = \int_{0}^{3} f(t)\,dt + \int_{3}^{15} f(t)\,dt$

$\left(3(7) + \left(\frac{3 \cdot 6}{2}\right)\right) + \left(12(1) + \frac{12(12)}{2}\right) = (21+9) + (12+72) = \boxed{114 \text{ meters}}$ ½ ½

[b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 15$ seconds using three subintervals and left endpoints.

NOTE: You must show the arithmetic expression that you used to get your answer. left endpoint $\lim_{n\to\infty} \sum_{i=0}^{n-1} f(a+i\Delta x)$

$\int_{0}^{3} f(t)\,dt + \int_{3}^{16} f(t)\,dt - \int_{15}^{16} f(t)\,dt$

$= 30 + \frac{13 \cdot 13}{2} - \frac{1}{2}$

$= 30 + \frac{168}{2} = 30 + 84 = \boxed{114 \text{ m}}$

$= \boxed{114 \text{ m}}$ ½

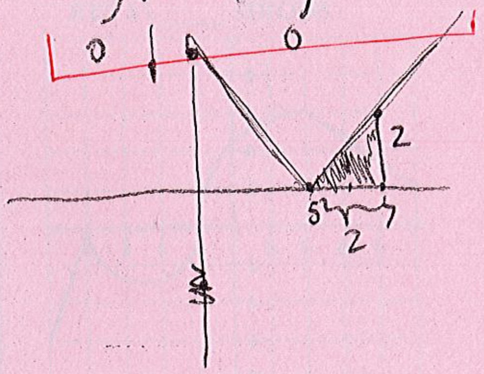
Evaluate $\int_0^7 (|x-5| - 4\sqrt{49-x^2})\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE:** 4 / 5 PTS

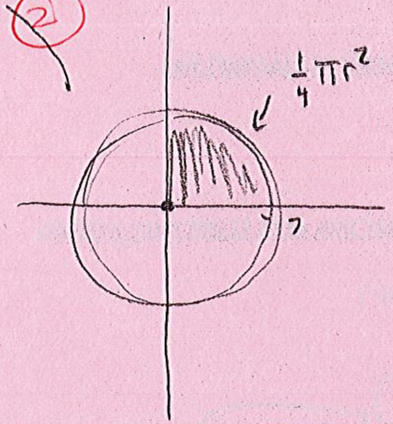
NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

line. $x^2+y^2=49$. circle center 0 radius 7.

$= \int_0^7 |x-5|\,dx + (-4)\int_0^7 \sqrt{49-x^2}\,dx$

0 0 (2)





~~$\frac{2(2)}{2} + \frac{1}{4}\pi r^2 \quad 2 + \frac{49\pi}{4} \quad 2 + 49\pi$~~

$\frac{1}{2}$

$= \frac{2(2)}{2} + (-4)\frac{49\pi}{4}$ $(1\frac{1}{2})$

$= \boxed{2 - 49\pi}$

Using the limit definition of the definite integral, and right endpoints, find $\int_{-1}^{5} (4x^2+8x)\,dx$. **SCORE:** $4\frac{1}{2}$ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$\lim_{n\to\infty} \Delta x \sum_{i=1}^{n} f(a + i\Delta x) \qquad \Delta x = \frac{6}{n}$

(1)

$= \lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} f(-1 + \frac{6i}{n})$

(1)

$= \lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} \left(4(-1+\frac{6i}{n})^2 + 8(-1+\frac{6i}{n})\right)$ $(1\frac{1}{2})$

$= \lim_{n\to\infty} \frac{6}{n} \sum_{i=1}^{n} \left(4(1 - \frac{12i}{n} + \frac{36i^2}{n^2}) + 8(-1+\frac{6i}{n})\right)$

$= \lim_{n\to\infty} \frac{24}{n} \sum_{i=1}^{n} \left(-1 + \frac{36i^2}{n^2}\right)$

$= \lim_{n\to\infty} \frac{24}{n} \left[\sum_{i=1}^{n} -1 + \sum_{i=1}^{n} \frac{36i^2}{n^2}\right]$

$= \lim_{n\to\infty} \frac{24}{n} \left(-1 + \frac{36}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right)$ (1)

$= \lim_{n\to\infty} \frac{-24}{n} + \frac{144(n+1)(2n+1)}{n^2}$

$= \lim_{n\to\infty} 144\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) - \frac{24}{n}$

$\lim_{n\to\infty} 144\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - \frac{24}{n}$

$= 144(1+0)(2+0) - 0$

$= \boxed{288}$

Spare work for problem #2

$\int_0^x f(t)\,dt \qquad \int_x^{15} f(t)\,dt$

$\int_0^x$

$\int_0^x$